

# Economics Lecture 11

2016-17

Sebastiano Vitali

# Course Outline

## 1 Consumer theory and its applications

1.1 Preferences and utility

1.2 Utility maximization and uncompensated demand

1.3 Expenditure minimization and compensated demand

1.4 Price changes and welfare

1.5 Labour supply, taxes and benefits

1.6 Saving and borrowing

## 2 Firms, costs and profit maximization

2.1 Firms and costs

2.2 Profit maximization and costs for a price taking firm

## 3. Industrial organization

3.1 Perfect competition and monopoly

3.2 Oligopoly and games

## 3.2 Oligopoly and Games

## 3.2 Oligopoly and Games

1. Introduction to Game Theory
2. Prisoner's dilemma
3. Dominant strategy & Cournot-Nash equilibria
4. Nash equilibrium
5. Bertrand-Nash equilibrium

6. Pure and mixed strategies
7. Multiple equilibria
8. Simultaneous (normal form) and sequential move (extensive form) games
9. Stackelberg equilibrium
10. Repeated games

# 1. Introduction to game theory

This is the way modern economists model oligopoly  
(industries with a small number of firms who take into account each others actions)

It is also used to model many other situations.

Language for game theory

Games have **players**.

Each player has a **strategy**.

**Payoffs** depend on strategies and are illustrated in the **payoff matrix**.

		Firm 2	
		large	small
Firm 1	large	16, 16	20, 15
	small	15, 20	18, 18

20, 15 in the top right box means player 1 gets 20 player 2 gets 15 when 1 plays large & 2 plays small.

The players are firm 1 and firm 2.

The players' strategies are large output & small output.

The payoff for each player depends on the choice of strategy by all players.

The table is the **payoff matrix**



If player 1 chooses large does player 2 choose large or small? 

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome?

What payoffs do the players get in this outcome?

Is there any way the players could get higher payoffs?

If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?



If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome?

What payoffs do the players get in this outcome?

Is there any way the players could get higher payoffs?

If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small? 

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome?

What payoffs do the players get in this outcome?

Is there any way the players could get higher payoffs?

If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small? 

What is the likely outcome?

What payoffs do the players get in this outcome?

Is there any way the players could get higher payoffs?

If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome? 

What payoffs do the players get in this outcome?

Is there any way the players could get higher payoffs?

If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome? **Both play large.**

What payoffs do the players get in this outcome?



Is there any way the players could get higher payoffs?

If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome? **Both play large.**

What payoffs do the players get in this outcome? **16, 16**

Is there any way the players could get higher payoffs?



If player 1 chooses large does player 2 choose large or small?

If player 1 chooses small does player 2 choose large or small?

If player 2 chooses large does player 1 choose large or small?

If player 2 chooses small does player 1 choose large or small?

What is the likely outcome? **Both play large.**

What payoffs do the players get in this outcome? **16, 16**

Is there any way the players could get higher payoffs?

**Yes both play small giving 18, 18**



# Economics Lesson on Cartels

Think of this as a model of a cartel.

Limiting production increases profits for all firms.

But each firm has an incentive to increase output.

Cartels are difficult to sustain.

Let's play!

# Prisoner's dilemma

## 2. Prisoner's dilemma

		Karel	
		confess	not confess
Vašek	confess	16, 16	20, 15
	not confess	15, 20	18, 18

Vašek and Karel are prisoners. They are being interrogated and are offered a reduction in prison sentence to anyone who confesses.

Both have an incentive to confess, but they both do worse if they both confess than they would do if neither confessed.

The Prisoner's dilemma game was formulated in the Cold War as a model of the nuclear arms race, the players were USA and the Soviet Union.

Many other situations as

- international trade negotiations
- international action on climate change
- ...

can all be modelled as prisoner's dilemma.

# Lessons from Game Theory

1. In the standard competitive model people acting **individually in their own** self interest achieve a **Pareto efficient** outcome.

In the prisoner's dilemma model the opposite happens, the outcome when both act in their **individual interest is worse** for both of them than if they act cooperatively.

2. In sport games are **zero sum**, one team's win is another team's loss.

Prisoners' dilemma is not zero sum.

**Life is not zero sum.**

# Cournot-Nash equilibrium

### 3. Dominant strategy & Cournot-Nash equilibria

A strategy is **dominant** if it maximizes a player's payoff whatever the other player does.

In prisoner's dilemma both players have a dominant strategy, confess.

The prisoner's dilemma has a **dominant strategy equilibrium**, i.e. a situation in which each player has, and plays, a dominant strategy.

Many games do not have a dominant strategy equilibrium, as in the following model.



## Definition of a Cournot-Nash equilibrium in a duopoly model

In the Cournot model of a duopoly (industry with 2 firms) each firm's strategy is its output.

In the **Cournot-Nash equilibrium** the outputs  $q_1$  and  $q_2$  have the property that

given  $q_2$  firm 1 maximizes its own profits by choosing  $q_1$ .

given  $q_1$  firm 2 maximizes its own profits by choosing  $q_2$ .

Demand is given by  $p = a - bQ$  where  $a > 0$  and  $b > 0$ .

If one firm produces  $q_1$  and the other produces  $q_2$  then industry quantity is  $Q = q_2 + q_1$ .

The firm producing  $q$  has total revenue

$$p q_1 = (a - b (q_1 + q_2)) q_1$$

The firm's profits are  $pq_1 - cq_1 = (a - c - b (q_1 + q_2)) q_1$ .

If  $a \leq c$  profits can't be positive for any positive  $q_1$  and  $q_2$  so the firm shuts down.

From here on assume  $a > c$ .

$$\text{Profits } p q_1 - c q_1 = (a - c - b (q_1 + q_2)) q_1$$

is a quadratic in  $q$  with negative coefficient for  $q_1^2$  so first order conditions give a maximum.

If maximum profits are negative the firm shuts down.

If  $a - c - bq_2 < 0$  the firm can't make profits with  $q_1 > 0$  so shuts down.

If  $a - c - bq_2 \geq 0$  profits are maximized where  $q$  satisfies foc

$$a - c - bq_2 - 2bq_1 = 0 \text{ so } q_1 = (a - c - bq_2) / 2b.$$

Profit max for firm 1 at

$$q_1 > 0 \text{ if } a - bq_2 - c \geq 0 \quad \& \quad a - 2bq_1 - bq_2 = c.$$

Similarly for firm 2 profits are maximized at

$$q_2 > 0 \text{ if } a - bq_1 - c \geq 0 \quad \& \quad a - 2bq_2 - bq_1 = c.$$

Solving simultaneously

$$a - 2bq_1 - bq_2 = c$$

$$a - 2bq_2 - bq_1 = c$$

$$\text{gives } q_1 = (a - c)/3b \quad q_2 = (a - c)/3b$$

$q_1$  and  $q_2$  both  $> 0$  because  $a > c$  and  $b > 0$ .

# Price, quantity and profits in the Cournot duopoly model

$$\text{Firm output } q_1 = (a - c)/3b \quad q_2 = (a - c)/3b$$

$$\text{Industry output } Q = q_1 + q_2 = 2/3 (a - c)/b$$

$$\text{Industry price } p = a - bQ = 1/3 a + 2/3 c = c + 1/3 (a - c)$$

$$\text{Profits for firm 1} = (p - c)q_1 = (a - c)^2/9b = \text{profits for firm 2.}$$

Profits are higher if costs are lower ( $c$  smaller), or if demand is higher ( $a$  bigger or  $b$  smaller)

## Reaction functions for the Cournot Model

We need to solve singularly the equations

$$a - 2bq_1 - bq_2 = c \quad \& \quad a - 2bq_2 - bq_1 = c$$

We get  $q_1$  as a function of  $q_2$ , i.e. 1's reaction function, giving its profit maximizing level of output given  $q_2$

$$q_1 = (a - c - bq_2)/2b.$$

We get  $q_2$  as a function of  $q_1$ , i.e. 2's reaction function, giving its profit maximizing level of output given  $q_1$

$$q_2 = (a - c - bq_1)/2b.$$

**in Cournot Nash equilibrium both firms are on their reaction functions.**

Starting from the reaction functions

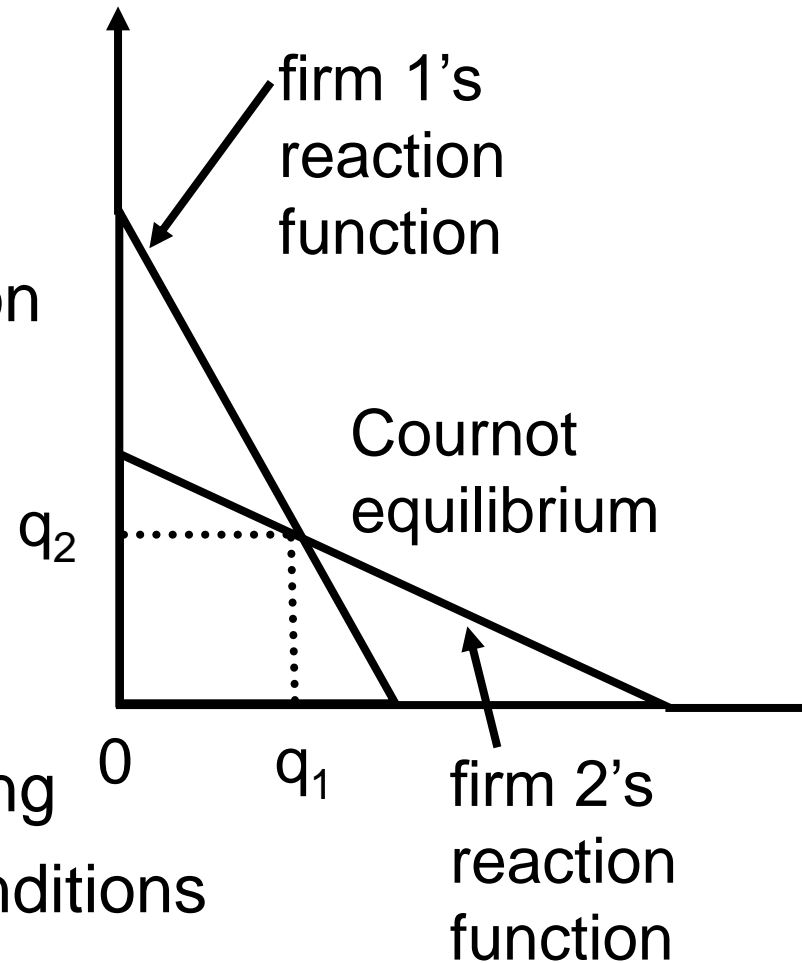
$$q_1 = \frac{(a - c - bq_2)}{2b}$$

$$q_2 = \frac{(a - c - bq_1)}{2b}$$

you can find  $q_1$  and  $q_2$  by substitution starting with

$$q_2 = \frac{\left[ a - c - b \left( \frac{a - c - bq_2}{2b} \right) \right]}{2b}.$$

This gives the same result as solving for  $q_1$  and  $q_2$  from the first order conditions but the algebra is harder.



# Nash equilibrium and dominant strategy equilibrium



## 4. Nash Equilibrium

Cournot Nash equilibrium is a special case of a Nash equilibrium.

In a Nash equilibrium each player's strategy maximizes his payoff, given the strategies pursued by the other players.

In a Cournot model the strategy is output.

# Nash equilibrium and dominant strategy equilibrium

In prisoner's dilemma confess is a dominant strategy, because it is the best thing to do whatever the other player do.

In a dominant strategy equilibrium each player has and plays a dominant strategy.

A dominant strategy equilibrium is always a Nash equilibrium.

But a Nash equilibrium is not always a dominant strategy equilibrium.

## 5. Bertrand Nash Equilibrium with 2 firms and identical goods

In a Bertrand duopoly game price is the strategic variable.

Suppose 2 firms produce an identical good and both have total cost  $cq$  so  $AC = MC = c$ .

If both firms charge the same price  $p$  they share the market equally.

If one firm charges a lower price it takes the entire market.

# Bertrand Nash Equilibrium

If firm 1 charges  $p_1 > c$ , firm 2's best response is to charge  $p_2 > c$  where  $p_2$  is just less than  $p_1$ .

Firm 1 sells nothing and makes 0 profits,

But firm 1 could do better by charging just less than  $p_2$ , so  $p_1, p_2$  is not a Nash equilibrium.

If either firm charges less than  $c$  it makes losses and could do better by charging  $c$ .

The Bertrand Nash equilibrium has  $p_1 = p_2 = c$ .

# Bertrand Nash Equilibrium with 2 firms producing goods that are imperfect substitutes

Two firms produce goods which are substitutes but not perfect substitutes

Demand for firm 1's output

$$q_1 = 2 - 3p_1 + 3p_2,$$

Demand for firm 2's output

$$q_2 = 6 - 2p_2 + p_1.$$

# Comparing Bertrand and Cournot

Assume there are 2 identical firms

both have cost total cost  $cq$ ,  $MC = AC = c$ .

In Bertrand Nash equilibrium  $p_1 = p_2 = c$ .

This is the same outcome as in perfect competition.

Each firm make 0 profits.

In Cournot Nash equilibrium if  $a > c$   $p = \frac{1}{3} a + \frac{2}{3} c > c$ ,

Each firm make profits  $(a - c)^2/9b > 0$ .

# Bertrand or Cournot?

Which model is appropriate depends on the real world situation you are trying to understand.

The result that price setting gives the same result as perfect competition does not hold if

- the goods the firms produce are not perfect substitutes
- or firms commit to quantity (capacity) before they set prices.

## n firm Cournot Model

Assume demand  $p = a - bQ$ , there are  $n$  firms each firm has total cost  $cq$  so  $MC = AC = c$

Using the same argument as with duopoly if the other firms produce a total of  $q_{n-1}$  a firm producing  $q_1$  makes profits  $(a - b q_{n-1} - c) q_1 - b q_1^2$ .

If  $a - b q_{n-1} - c > 0$  profits are maximized at  $q_1 = \frac{1}{2} (a - b q_{n-1} - c)/b$ .

If all firms produce the same output  $q$  so  $q_{n-1} = (n-1)q$  these conditions are satisfied if  $a > c$  and

$$q_1 = \frac{(a - c)}{(n + 1)b}$$



$$\text{firm output } q = \frac{(a - c)}{(n + 1)b}$$

$$\text{industry output } Q = nq = \frac{n(a - c)}{(n + 1)b}$$

$$\text{price } p = a - bQ = a - \frac{n(a - c)}{(n + 1)} = c + \frac{(a - c)}{(n + 1)}$$

$$\text{each firm makes profits } (p - c)q = \frac{(a - c)^2}{(n + 1)^2 b}$$

# Pure and mixed strategies

## 6. Pure and mixed strategies

An enforcement game - find the Nash equilibrium

		car driver	
		park legally	park illegally
police officer	control	- 5, -10	15, - 100
	not control	0, - 10	0, 0

if the police officer controls, the driver parks

if the driver parks legally, the police officer

if the police officer does not control, the driver parks

if the driver parks illegally, the police officer

# An enforcement game find the Nash equilibrium

		car driver	
		park legally	park illegally
police officer	control	- 5, -10	<del>15, -100</del>
	not control	0, - 10	0, 0

if the police officer controls, the driver parks **legally**

if the driver parks legally, the police officer

if the police officer does not control, the drive

if the driver parks illegally, the police officer

# An enforcement game

## find the Nash equilibrium

		car driver	
		park legally	park illegally
police officer	control	<del>- 5, -10</del>	<del>15, -100</del>
	not control	0, - 10	0, 0

if the police officer controls, the driver parks **legally**

if the driver parks legally, the police officer **does not control**

if the police officer does not control, the driver parks

if the driver parks illegally, the police officer

# An enforcement game find the Nash equilibrium

		car driver	
		park legally	park illegally
police officer	control	<del>-5, -10</del>	<del>15, -100</del>
	not control	<del>0, -10</del>	0, 0

if the police officer controls, the driver parks **legally**

if the driver parks legally, the police officer **does not control**

if the police officer does not control, the driver parks **illegally**

if the driver parks illegally, the police officer

# An enforcement game find the Nash equilibrium

		car driver	
		park legally	park illegally
police officer	control	<del>-5, -10</del>	<del>15, -100</del>
	not control	<del>0, -10</del>	<del>0, 0</del>

none of the cells is a Nash equilibrium

if the police officer controls, the driver parks **legally**

if the driver parks legally, the police officer **does not control**

if the police officer does not control, the driver parks **illegally**

if the driver parks illegally, the police officer **controls**

# Pure and mixed strategies

A player plays a **pure strategy** if she does not randomize, e.g. she always controls, or always doesn't control.

A player plays a **mixed strategy** if she randomizes, e.g. she controls with probability  $1/3$  and doesn't control with probability  $2/3$ .

The enforcement game has no equilibrium in pure strategies.



# The enforcement game has an equilibrium in mixed strategies

If the driver believes that the police office controls

with probability  $w$ , then the driver's expected payoff from

legal parking

$$= -10w - 10(1 - w)$$

The driver's expected payoff from illegal parking

$$= -100w + 0(1 - w).$$

# The enforcement game has an equilibrium in mixed strategies

The driver is indifferent between parking legally and parking illegally and is willing to randomize if

$$-10w - 10(1 - w) = -100w + 0(1 - w)$$

that is if  $w = 0.1$

# The enforcement game has an equilibrium in mixed strategies

If the police officer believes driver parks legally with probability  $d$

Expected payoff from controlling =  $-5d + 15(1 - d)$

Expected payoff not controlling =  $0d + 0(1 - d)$

The police officer is indifferent between controlling and not controlling parking and is willing to randomize if

$$-5d + 15(1 - d) = 0d + 0(1 - d)$$

that is  $d = 0.75$

# The enforcement game has an equilibrium in mixed strategies

This game has an equilibrium in mixed strategies where the warden patrols with probability 0.1 & the driver parks legally with probability 0.75.

Note: in the equilibrium in an mixed strategies, then

- the probability that the police officer controls is determined by the driver's indifference condition
- the probability that the driver parks legally is determined by the police officer's indifference condition

# Existence Question (Not for this course)

Do all games have an equilibrium in either pure or mixed strategies?

Yes, if the game has simultaneous moves and there are a finite number of players who each have a finite number of pure strategies.

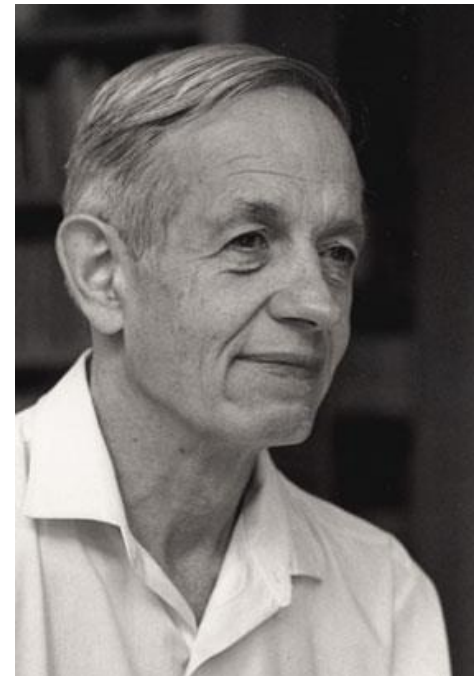
Result proved by Nash (1950).

Nobel Prize for Economics 1994

Biography

Sylvia Nasar, *A Beautiful Mind*

Faber and Faber 2002



# Lessons from Game Theory

There are games in which there is no equilibrium in pure strategies, so in the model players must randomize.

Examples, enforcement (e.g. parking, tax audit)

Sports, hit right or left randomly to keep opponent guessing.

# Multiple equilibria

# 7. Multiple equilibria

The computer choice game

		economist			
		Mac		PC	
biologist	Mac	2, 1	0, 0		
	PC	0, 0	1, 2		

Does this game have a Nash equilibrium in pure strategies?



# 7. Multiple equilibria

The computer choice game

		economist	
		Mac	PC
biologist	Mac	2, 1	0, 0
	PC	0, 0	1, 2

Does this game have a Nash equilibrium in pure strategies?

2 equilibria in pure strategies.

For you to work out, does it have an equilibrium in mixed strategies?

# Lessons from Game Theory

Games can have multiple equilibria.

So a game theoretic model may not give a prediction of the outcome.

This is especially true if the same players play many times.

The outcomes of game theoretic model are very sensitive to the assumptions of the model, so similar models may give very different predictions.

# Modelling entry to an industry as a game

	incumbent = firm already in industry		
		fight if there is entry	not fight if there is entry
potential entrant = firm deciding whether to enter	not enter	0, 9	0, 9
	enter	-1, 0	1, 1

Nash equilibrium?

# Modelling entry to an industry as a game

		incumbent = firm already in industry	
		fight if there is entry	not fight if there is entry
potential entrant = firm deciding whether to enter	not enter	0, 9	0, 9
	enter	-1, 0	1, 1

Nash equilibrium?

is the threat to fight if there is entry credible?

# Simultaneous and sequential move games

## 8. Simultaneous and sequential move games

So far we have assumed that players choose their strategies simultaneously and used a payoff matrix to illustrate the game.

Games like this are called **simultaneous move** games (also **normal form** games).

Entry is better modelled as **sequential move game** (sometimes called an **extensive form** game).

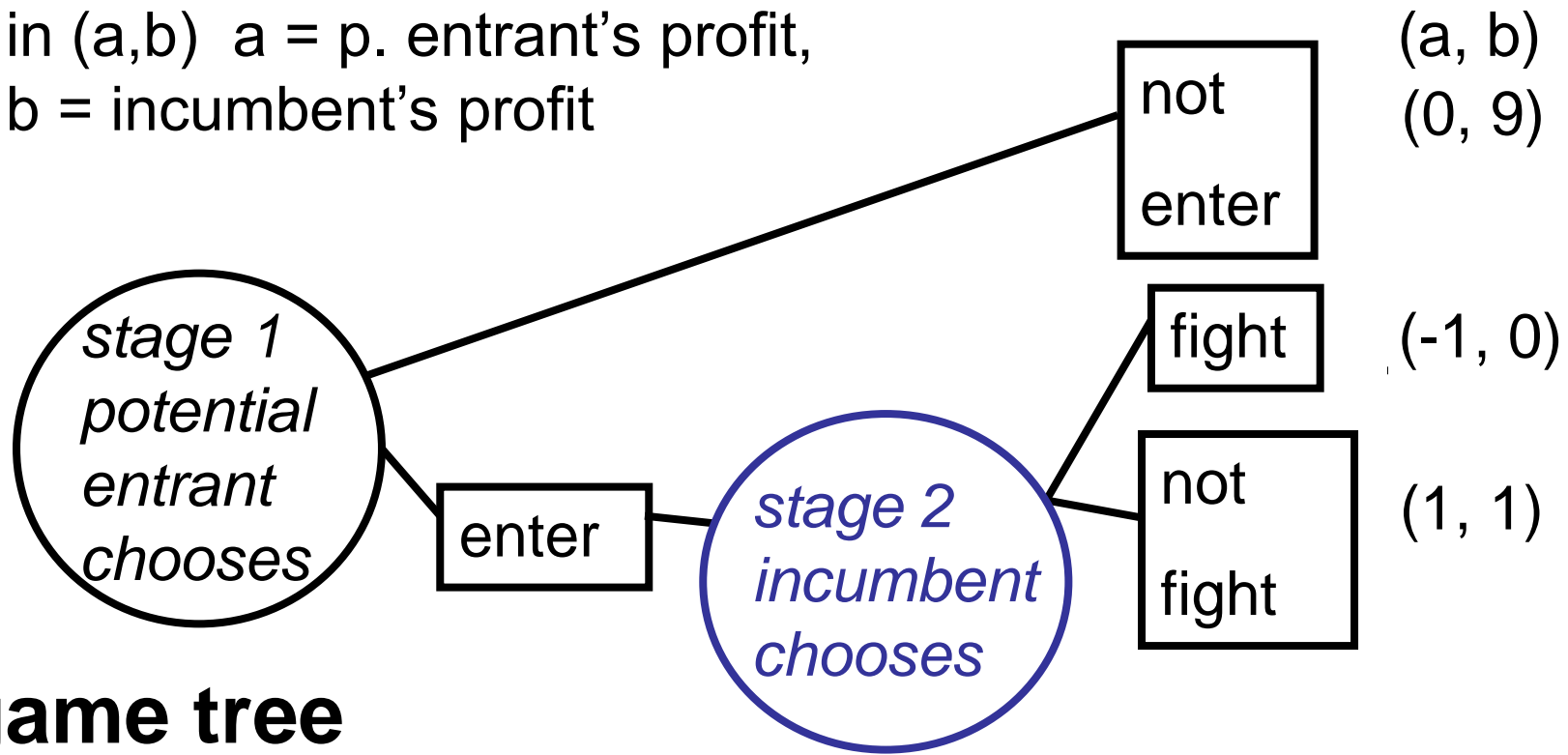
This simple game has 2 stages.

stage 1 potential entrant chooses whether to enter

stage 2 incumbent chooses whether to fight.

Extensive form games are analysed using a game tree.

in  $(a,b)$   $a = p.$  entrant's profit,  
 $b =$  incumbent's profit



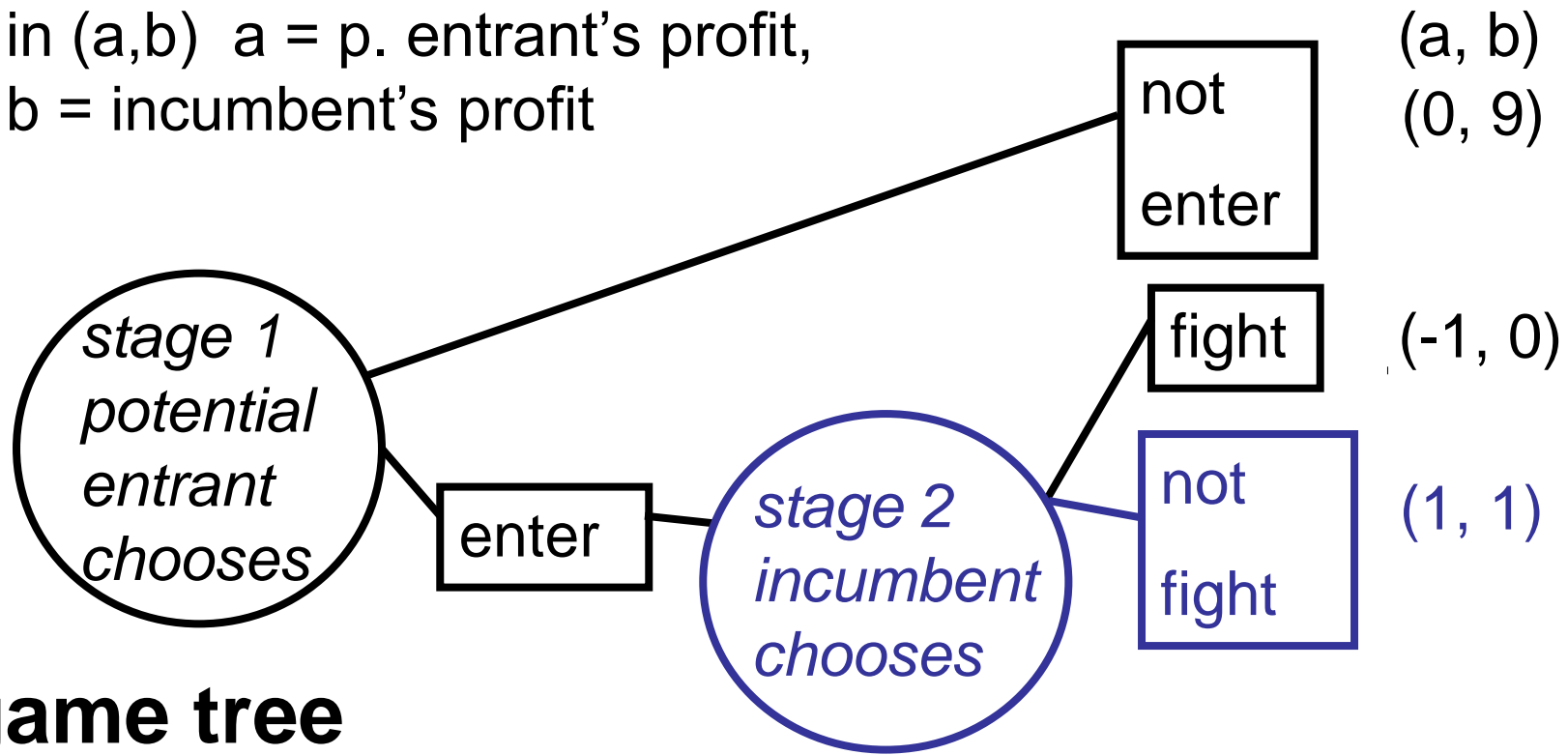
## game tree

If the potential entrant does not enter the incumbent has no choice to make. Potential entrant gets 0, incumbent 9.

If the potential entrant enters at stage 1 what does the incumbent do at stage 2?

What does the potential entrant do at stage 1?

in  $(a,b)$   $a = p.$  entrant's profit,  
 $b =$  incumbent's profit



## game tree

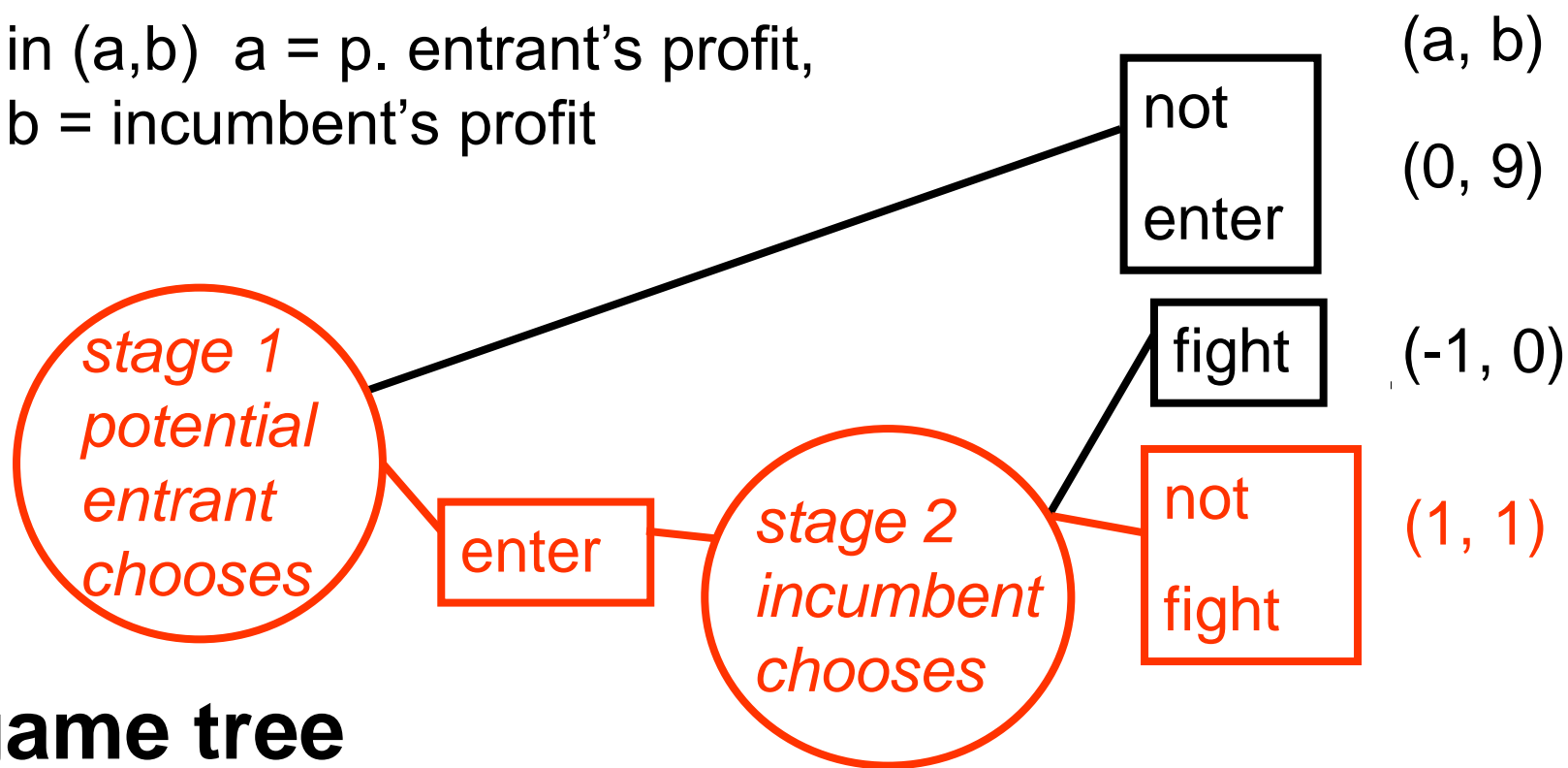
If the potential entrant does not enter the incumbent has no choice to make. Potential entrant gets 0, incumbent 9.

If the potential entrant enters at stage 1 what does the incumbent do at stage 2? **Does not fight because gets 0 if fights, 1 if not fight.**

What does the potential entrant do at stage 1?



in  $(a,b)$   $a = p.$  entrant's profit,  
 $b =$  incumbent's profit



## game tree

If the potential entrant does not enter the incumbent has no choice to make. Potential entrant gets 0, incumbent 9.

If the potential entrant enters at stage 1 what does the incumbent do at stage 2? **Does not fight because gets 0 if fights, 1 if not fight.**

What does the potential entrant do at stage 1?

**Enters because will get 1 if enters 0 if doesn't enter.**

In the entry game the incumbent would like to commit to fighting if there is entry so as to deter entry.

But the commitment is not credible because once there is entry it is not profitable to fight it.

Commitment can be strategically useful.

Commitment strategies,

in the entry game investing in capacity to reduce marginal cost.

when invading, burning boats

somehow reducing the payoff to not fighting.

The entry game looked at as a simultaneous move game has two Nash equilibria (not enter, fight) & (enter, not fight).

Looking at this as a sequential move game it has one equilibrium, (enter not fight).

Always analyse sequential move games by **backward induction**.

What does last player to move do, given what other players have already done?

What does the next to last player to move, given what other players have already done, and knowing how last player to move will respond?

.....

What does the first player to move do, knowing how players will respond in all future moves?

# Stackelberg equilibrium

## 9. Stackelberg equilibrium

There are two firms 1 (leader) and 2 (follower) with costs

$$cq_1, cq_2$$

$$p = a - b(q_1 + q_2).$$

Cournot assumes  $q_1$  and  $q_2$  are chosen simultaneously.

Stackelberg assumes 2 stages.

Stage 1 leader chooses  $q_1$ .

Stage 2 follower chooses  $q_2$ .

At stage 2 the follower chooses  $q_2$  taking  $q_1$  as given.

The follower's profits are maximized by setting.

$$q_2 = \frac{(a - c - bq_1)}{2b}$$

At stage 1 the leader chooses  $q_1$  taking into account how the follower will respond when choosing  $q_2$ .

The leader's profits are

$$\begin{aligned} (p - c)q_1 &= (a - c - b(q_1 + q_2))q_1 \\ &= \left\{ a - c - b \left[ q_1 + \left( \frac{a - c - bq_1}{2b} \right) \right] \right\} q_1 = \left\{ \frac{1}{2} (a - c) - \frac{1}{2} bq_1 \right\} q_1 \end{aligned}$$

which are maximized by setting  $q_1 = \frac{a - c}{2b}$  at stage 1.

$$\text{As } q_2 = \frac{(a - c - bq_1)}{2b} \text{ and } q_1 = \frac{a - c}{2b}$$

$$q_2 = \frac{(a - c)}{4b} \text{ price } p = a - b(q_1 + q_2) = \frac{a}{4} + \frac{3c}{4}$$

The leader's profits are  $(p - c)q_1 = (a - c)^2 / 8b$ .

The follower's profits are  $(p - c)q_2 = (a - c)^2 / 16b$ .

**Summary Table !!!**



Firm cost = $cq$ ( $q$ firm output)    price $p = a - bQ$ ( $Q$ industry output)					
	price	firm output	industry output	firm profits	industry profits
perfect competition	$c$	un-determined	$\frac{(a - c)}{b}$	$0$	$0$
$n$ firm Cournot-Nash	$c + \frac{(a - c)}{(n+1)}$	$\frac{(a - c)}{(n+1)b}$	$\frac{n(a - c)}{(n+1)b}$	$\frac{(a - c)^2}{(n+1)^2b}$	$\frac{n(a - c)^2}{(n+1)^2b}$
2 firm Cournot-Nash	$c + \frac{(a - c)}{3}$	$\frac{(a - c)}{3b}$	$\frac{2(a - c)}{3b}$	$\frac{(a - c)^2}{9b}$	$\frac{2(a - c)^2}{9b}$
Stackelberg	$c + \frac{(a - c)}{4}$	leader $\frac{(a - c)}{2b}$ follower $\frac{(a - c)}{4b}$	$\frac{3(a - c)}{4b}$	leader $\frac{(a - c)^2}{8b}$ follower $\frac{(a - c)^2}{16b}$	$\frac{3(a - c)^2}{16b}$
monopoly	$c + \frac{(a - c)}{2}$	$\frac{(a - c)}{2b}$	$\frac{(a - c)}{2b}$	$\frac{(a - c)^2}{4b}$	$\frac{(a - c)^2}{4b}$

# Comparisons

Price and industry profits are highest in monopoly and lowest with perfect competition.

As  $n$ , the number of firms in a Cournot-Nash model, gets larger, price falls, industry output increases, industry profits fall.

When  $n$  is very large price, industry output and industry profits are close to their perfect competition levels.

Comparing Stackelberg and 2 firm Cournot-Nash, in Stackelberg price is lower, industry profits are lower, leader's profits are higher, follower's lower than in C N.

Let's play!

# Repeated games

just think about the games:  
learn from the experience

# What have we achieved?

- Modelling of simple strategic interactions
- Oligopoly
- No
  - product differentiation
  - R&D and innovation
  - Advertising – digital & other
  - etc., etc.
  - can be modelled.